NAG Toolbox for MATLAB

s18ac

1 Purpose

s18ac returns the value of the modified Bessel Function $K_0(x)$, via the function name.

2 Syntax

```
[result, ifail] = s18ac(x)
```

3 Description

s18ac evaluates an approximation to the modified Bessel Function of the second kind $K_0(x)$.

Note: $K_0(x)$ is undefined for $x \leq 0$ and the function will fail for such arguments.

The function is based on five Chebyshev expansions:

For $0 < x \leq 1$,

$$K_0(x) = -\ln x \sum_{r=0}{}' a_r T_r(t) + \sum_{r=0}{}' b_r T_r(t), \qquad \text{where } t = 2x^2 - 1.$$

For $1 < x \leq 2$,

$$K_0(x) = e^{-x} \sum_{r=0}{}' c_r T_r(t), \qquad \text{where } t = 2x - 3.$$

For $2 < x \leq 4$,

$$K_0(x) = e^{-x} \sum_{r=0}{}' d_r T_r(t), \qquad \text{where } t = x - 3.$$

For $x > 4$,

$$K_0(x) = \frac{e^{-x}}{\sqrt{x}} \sum_{r=0}{}' e_r T_r(t), \text{where } t = \frac{9 - x}{1 + x}.$$

For x near zero, $K_0(x) \simeq -\gamma - \ln\left(\frac{x}{2}\right)$, where γ denotes Euler's constant. This approximation is used when x is sufficiently small for the result to be correct to ***machine precision***.

For large x, where there is a danger of underflow due to the smallness of K_0, the result is set exactly to zero.

4 References

Abramowitz M and Stegun I A 1972 *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

5 Parameters

5.1 Compulsory Input Parameters

1: **x – double scalar**

The argument x of the function.

Constraint: **x** > 0.0.

5.2 Optional Input Parameters

None.

5.3 Input Parameters Omitted from the MATLAB Interface

None.

5.4 Output Parameters

1: **result – double scalar**

The result of the function.

2: **ifail – int32 scalar**

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= 1$

x ≤ 0.0, K_0 is undefined. On soft failure the function returns zero.

7 Accuracy

Let δ and ϵ be the relative errors in the argument and result respectively.

If δ is somewhat larger than the ***machine precision*** (i.e., if δ is due to data errors etc.), then ϵ and δ are approximately related by:

$$\epsilon \simeq \left|\frac{xK_1(x)}{K_0(x)}\right|\delta.$$

Figure 1 shows the behaviour of the error amplification factor

$$\left|\frac{xK_1(x)}{K_0(x)}\right|.$$

However, if δ is of the same order as ***machine precision***, then rounding errors could make ϵ slightly larger than the above relation predicts.

For small x, the amplification factor is approximately $\left|\frac{1}{\ln x}\right|$, which implies strong attenuation of the error, but in general ϵ can never be less than the ***machine precision***.

For large x, $\epsilon \simeq x\delta$ and we have strong amplification of the relative error. Eventually K_0, which is asymptotically given by $\frac{e^{-x}}{\sqrt{x}}$, becomes so small that it cannot be calculated without underflow and hence the function will return zero. Note that for large x the errors will be dominated by those of the standard function EXP.

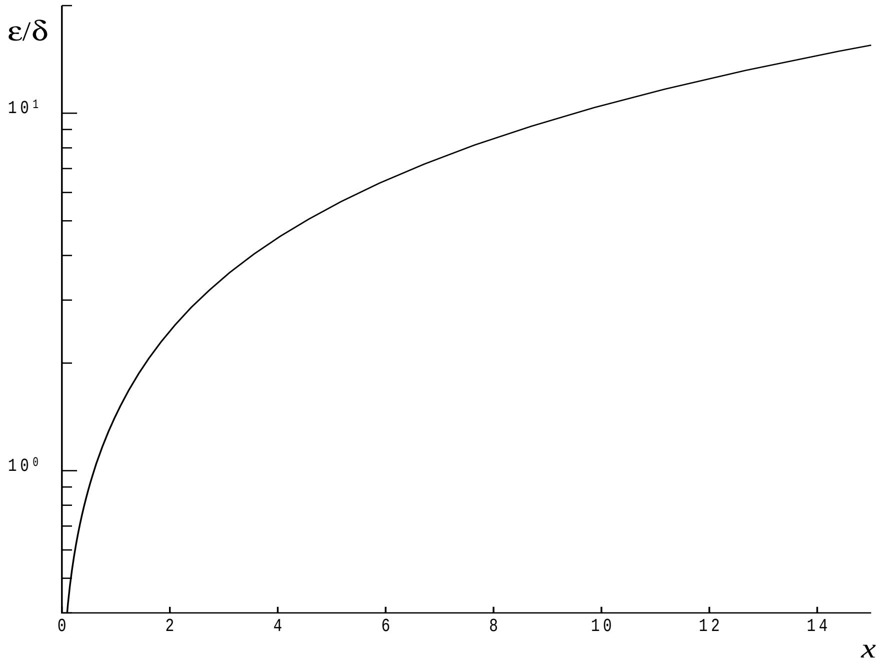

Figure 1

8 Further Comments

None.

9 Example

```
x = 0.4;
[result, ifail] = s18ac(x)
```

```
result =
    1.1145
ifail =
           0
```